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Gauge Enhancement and Chirality Changes in Nonperturbative Orbifold Models

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Consistent heterotic orbifold vacua, including nonperturbative information, can be constructed. Generically, the usual modular invariance constraints are violated and thus, anomalies are expected. However, five-branes do appear in the correct proportion just to ensure the consistency of the full theory. We first exemplify the situation in six dimensions, $N = 1$, where strong-coupling effects, implying the presence of five-branes, are better known. Explicit $N = 1$ models in $D = 4$, essentially inherited from six dimensions, are then presented. In particular we show concrete examples of models which exhibit nonperturbative transitions leading to gauge enhancement and/or to a change in the number of chiral generations.

1 INTRODUCTION

In this paper, based on work of ref. 1, we deal with nonperturbative vacua in heterotic orbifold compactifications. In particular we present explicit realizations of transitions among different vacua, in four dimensions, where the gauge group gets enhanced and/or a change in the net number of fermionic generations occurs. Let us stress that such kinds of phenomena are not possible in perturbation theory. For instance, the number of generations is given by the index of the Dirac operator on the compactified manifold and is therefore a topological invariant.

Due to their simplicity, perturbative heterotic orbifolds [2, 3] have proven to be a very powerful tool in building “semirealistic-stringy inspired,” effective low-energy models (Standard Model-like, higher level StrinGuts, etc.). Not only can the gauge group and matter multiplets be easily obtained (and somewhat controlled), but the structure of Yukawa couplings and symmetry-

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breaking patterns can be easily studied, etc. The full partition function may be constructed in these cases.

We will not be able to go that far in the nonperturbative case and we will concentrate on the structure of the massless spectrum.

Our first aim will be to incorporate nonperturbative contributions in six-dimensional orbifold compactifications of the heterotic, both $E_8 \times E_8$ and $SO(32)$, string theories. If nonperturbative phenomena are to be included, this seems a good road before descending to the more involved and less known four-dimensional case. In fact, we will show how relevant information in four dimensions may be obtained from six.

The case $D = 6$ is undoubtedly interesting in itself. $N = 1$ theories are chiral, and consistency, that is, anomaly cancellation, constrains the allowed theories severely. Moreover, even if still incomplete, many nonperturbative effects are quite well understood in six dimensions. In particular, examples have been derived from different approaches as type IIB orientifolds F-theory and M-theory.

In an orbifold compactification Z_M symmetry is divided out. Acting on the (complex) bosonic transverse coordinates, the Z_M twist generator θ has eigenvalues $e^{2\pi i v_a}$. In $D = 6$, v_a are the components of $v = (0, 0, 1/M, -1/M)$ and M can take the values $M = 2, 3, 4, 6$. The embedding of θ on the gauge degrees of freedom is usually performed by a shift V such that MV belongs to the $E_8 \times E_8$ lattice $\Gamma_8 \times \Gamma_8$ or to the $Spin(32)/Z_2$ lattice Γ_{16}.

In perturbative string theory, this shift is restricted by the modular invariance constraint

$$M(V^2 - v^2) = \text{even} \tag{1.1}$$

The spectrum for each model is subdivided into sectors. There are M sectors twisted by θ^j, $j = 0, 1, \ldots, M - 1$. Each particle state is created by a product of left and right vertex operators $L \otimes R$. At a generic point in the four-torus moduli space, the massless states follow from

$$\begin{aligned} m_R^2 &= N_R + \tfrac{1}{2}(r + jv)^2 + E_j - \tfrac{1}{2} \\ m_L^2 &= N_L + \tfrac{1}{2}(P + jV)^2 + E_j - 1 \end{aligned} \tag{1.2}$$

Here r is an $SO(8)$ weight with $\Sigma_{i=1}^{4} r_i = \text{odd}$ and P is a gauge lattice vector with $\Sigma_{I=1}^{16} P^I = \text{even}$. E_j is the twisted oscillator contribution to the zero-point energy and is given by $E_j = j(M - j)/M^2$. The multiplicity of states satisfying Eq. (1.2) in a θ^j sector is given by the appropriate generalized GSO projections [4, 5]. The gravity multiplet, a tensor multiplet, charged hypermultiplets, and two neutral hypermultiplets (four in the case of Z_2) appear in the untwisted sector. Twisted sectors contain only charged hypermultiplets. The generalized GSO projections are particularly simple in the Z_2 and Z_3 cases since all massless states survive with the same multiplicity.

Let us come back to Eq. (1.1). This constraint ensures level matching. It corresponds to an orbifold version of the global consistency of the theory ensuring anomaly cancellation. This consistency may be understood as the vanishing of the total magnetic charge associated with the antisymmetric tensor field. Namely, $\int_X dH = \int_X (F^2 - R^2) = 0$, where H is the three-form heterotic field strength with $dH = tr\, F^2 - tr\, R^2$ and X is the compact space. For an orbifold $X = T^4/Z_M$ and, since curvature is localized at Z_M the fixed points, we can restate this equation as

$$Q_{\mathrm{TOT}} = \sum_f Q_f = 0 \tag{1.3}$$

where integrals are taken around fixed points f. Equivalently, since the total Euler number of X is $\int_X R^2 = 24$ we have

$$I_{\mathrm{TOT}} = \sum_f I_f = 24 \tag{1.4}$$

where I_f is the instanton number at the fixed point.

The issue we want to stress here is that modular invariance requirement as stated in (1.1) and anomaly cancellation are equivalent. They are satisfied if there are 24 instantons at fixed points or equivalently if the magnetic charges at fixed points add up to zero. More explicitly, in ref. 1 it is shown [for the $SO(32)$ case] that $I_f = l + M\, E_\Theta$.

Here $E_\Theta = \Sigma_{I=1}^{16} \frac{1}{2} V_I(V_I - 1)$, $E_\theta = (M - 1)^2/M$, l is an integer, and V_I are the components of the shift V. Also, by computing the curvature at the fixed orbifold point it is found that

$$Q_f = l' + M(E_\Theta - E_\theta) \tag{1.5}$$

with $l' = l - M - 1$. Thus, for $Q_f = 0$, Eq. (1.1) is obtained.

Z_M orbifolds corresponding to all possible embeddings allowed by Eq. (1.1) can be constructed. Indeed, their corresponding massless spectra may be reproduced by application of index theorems on orbifold (ALE) singularities [1] with $I_{\mathrm{TOT}} = 24$ instantons.

The question to address now is: Could we still have a consistent theory if $I_{\mathrm{TOT}} < 24$ is allowed, i.e., when $n_B = 24 - I_{\mathrm{TOT}}$ instantons become small? Since the dilaton is known to diverge [6] in such a situation, nonperturbative information is required to answer this question. In fact, small instantons in both $SO(32)$ or $E_8 \times E_8$ have been studied [7, 8] and may be identified as five-branes, i.e., extended objects with their world volume filling six-dimensional spacetime. They correspond to type I *D5-branes* in the $SO(32)$ case and to M-theory five-branes for $E_8 \times E_8$, Five-branes act as magnetic sources for the antisymmetric tensor field and therefore Eq. (1.3) must now read

$$Q_{\mathrm{TOT}} = \sum_f Q_f + n_B = 0 \tag{1.6}$$

We see from our discussion of Eqs. (1.6) and (1.4), that when five-branes are present the "modular invariance" constraint on the shift V must be abandoned. This is certainly troublesome in perturbation theory, since only shifts complying with this constraint ensure anomaly cancellation. Other V's would lead to anomalous spectra. On the other hand, this should not be surprising when dealing with strong-coupling effects, since modular invariance is a perturbative concept (associated with the expansion in terms Riemann surfaces spanned in string propagation).

Generically (we will be more precise about this), these five-branes are expected to carry vector, hyper, and tensor massless (six-dimensional) multiplets on their world volumes and therefore they should contribute to the total, gauge, and gravitational anomaly of the spectrum.

All these elements suggest a possible positive answer to the above question. Perturbative contributions associated with "fat" I instantons and with n_B five-branes, with $I + n_B = 24$, would contribute to the massless spectrum such that the whole anomaly could cancel. In fact, we will see that this appears to be the case for the situations where nonperturbative information is at hand.

Let us first discuss the perturbative contribution to the massless spectrum. This spectrum corresponds to a number $I_{\mathrm{TOT}} < 24$ of large instantons. As indicated, the instanton number is a function of the shift V in the gauge lattice. This V has to comply with a new constraint depending on the number of five-branes since $\Sigma\ I_f(V) + n_B = 24$. For instance, assume that we have the same charge at each fixed point (this is expected for a Z_3 orbifold where all points are equivalent). Equation (1.6) tells us that $Q_f = -n_B/n_f$ where n_f is the number of fixed points. Following the steps that lead us to (1.5), we now obtain

$$M(V^2 - v^2) + 2ME_B(f) = \text{even} \tag{1.7}$$

where we have defined, for further convenience, $E_B(f) = -Mn_B/2n_f$. This gives us the result we expected. Moreover, recalling that (1.1) results by imposing level matching, our result suggests that masses of states could be obtained as in ordinary perturbative orbifolds by just modifying the mass of the left sector states to be

$$m_L^2 = N_L \tfrac{1}{2}(P + jV)^2 + E_j + E_B(j) - 1 \tag{1.8}$$

In fact, $m_L^2 = m_R^2$ leads to (1.7) with f a fixed point in twisted sector (j). We will propose this expression for computing the massless states in the perturbative sector of general orbifold models containing five-branes. E_B is interpreted as the shift in the vacuum energy due to the flux of the antisymmetric field. Since in general there will be nonequivalent fixed points we do not expect in general a simple relationship as above between this energy shift and n_B. The untwisted sector is obtained by projecting onto invariant states as usual.

In order to illustrate how this proposal works, let us consider the case of smooth Z_3 compactifications. This is the simplest case. There is just one θ twisted sector with an energy shift to be considered and nine equivalent fixed points. Smooth compactification means that oscillator modes needed to blow up orbifold singularities should be present, thus $N_L = 1/3$ in (1.8). For these modes (two at each fixed point) to be massless it is required that

$$V^2 = \tfrac{8}{9} - 2E_B \tag{1.9}$$

Thus the maximum shift in the vacuum energy will correspond to $E_B = 4/9$ (obtained for $V = 0$). The other extreme case is $V^2 = 8/9$, in which we have $E_B = 0$ corresponding to some modular invariant (perturbative) models.

Let us consider first the $SO(32)$ heterotic string with the class of shifts V with $3V \in \Gamma_{16}$ of the form

$$V = \tfrac{1}{3}(1, \ldots, 1, 0, \ldots, 0) \tag{1.10}$$

and $m \leq 8$. The unbroken group is $U(m) \times SO(32 - 2m)$ and the untwisted sector contains hypermultiplets transforming as $(\mathbf{m}, \mathbf{32 - 2m}) + (\mathbf{m(m - 1)/2}, \mathbf{1}) + 2(\mathbf{1}, \mathbf{1})$. The twisted sectors need an extra vacuum shift $E_B = (8 - m)/18$ and the mass formula provides massless hypermultiplets in each twisted sector transforming as

$$\left(\frac{\mathbf{m(m-1)}}{\mathbf{2}}, \mathbf{1}\right) + 2(\mathbf{1}, \mathbf{1}) \tag{1.11}$$

for $m = 0, 2, 4, 6, 8$.

There are two other Z_3 models with singlet moduli in the twisted sector. One of them, with shift $V = (2/3, 0, \ldots, 0)$, has gauge group $SO(30) \times U(1)$. The other model has shift $V = \tfrac{1}{6}(1, \ldots, 1)$, $3V$ being a spinorial weight. The gauge group is $U(16)$. It is thus a $SO(32)$ embedding without vector structure, a Z_3 analogue to the Z_2 orientifolds constructed in refs. 9 and 10.

Except for the $m = 8$ case, the remaining models, as they stand, have gauge and gravitational anomalies and the corresponding shifts do not fulfill the perturbative modular invariance constraints. However, it turns out that the addition of an appropriate number of five-branes renders them consistent. Indeed, one can check that adding $3(8 - m)$ five-branes to the vacua in Eq.

(1.10) (12 five-branes in the other two cases) leads to anomaly-free results. The case of five-branes or small $SO(32)$ instantons was considered in ref. 7. When n_B-branes coincide at the same point (and away from singularities) a nonperturbative gauge group $Sp(n_B)$ is expected to appear, along with hypermultiplets transforming in the fundamental, antisymmetric, and singlet representations. We also assign these hypermultiplets to representations of the perturbative group. Thus, the massless matter content transforming under the full $U(m) \times SO(32 - 2m) \times Sp(n_B)$ group is

$$\frac{1}{2}(\mathbf{m}, \mathbf{1}, \mathbf{2n_B}) + \frac{1}{2}(\overline{\mathbf{m}}, \mathbf{1}, \mathbf{2n_B}) + \frac{1}{2}(\mathbf{1}, \mathbf{32 - 2m}, \mathbf{2n_B}) + \left(\mathbf{1}, \mathbf{1}, \frac{\mathbf{2n_B(2n_B - 1)}}{\mathbf{2}} - \mathbf{1}\right) + (\mathbf{1}, \mathbf{1}, \mathbf{1}) \qquad (1.12)$$

It is straightforward to check that all non-Abelian gauge and gravitational anomalies do cancel. Thus, our construction provides a new class of consistent nonperturbative orbifold heterotic vacua.

Notice that the models obtained require the addition of $6s$, $s = 4, 3, 2, 1, 0$, five-branes. They contribute one unit of magnetic charge each. Thus, in order to achieve overall vanishing magnetic charge, each of the fixed points (which in these particular models are identical) must carry magnetic charge $q_f = n_B/9$.

The $E_8 \times E_8$ case is to some extent similar, but has some peculiarities. Consider the class of models generated by gauge shifts of the form

$$V = \tfrac{1}{3}(1, \ldots, 1, 0, \ldots, 0) \times \tfrac{1}{3}(1, \ldots, 1, 0, \ldots, 0) \qquad (1.13)$$

with an even number m_1 (m_2) of $\frac{1}{3}$ entries in the first (second) E_8 and with $m = m_1 + m_2 \leq 8$. Models with appropriate oscillator moduli in the twisted sector have $(m_1, m_2) = (0, 0), (2, 0), (4, 0), (2, 2), (2, 4)$, and $(4, 4)$. Again, none of these models [except for $(m_1, m_2) = (4, 4)$] fulfills the perturbative modular invariance constraints and they are therefore anomalous. However, unlike the $SO(32)$ case, they *do not present non-Abelian gauge anomalies*. We can check that they miss an equivalent of $3(8 - m) \times 30$ hypermultiplets in order to cancel gravitational anomalies. But this is precisely the contribution corresponding to $3(8 - m)$ M-theory five-branes, each one carrying a tensor multiplet and a gauge singlet hypermultiplets. Therefore, these missing modes match the nonperturbative spectrum corresponding to setting this same number of instantons to zero size in $E_8 \times E_8$. This is a nice check of our procedure. Simple addition of a shift in the vacuum energy automatically takes into account the difference between the $SO(32)$ and $E_8 \times E_8$ heterotic strings, yielding no gauge anomalies in the second case. The Z_3 models under consideration are orbifold realizations of the $E_8 \times E_8$ vacua in the presence of wandering branes considered in refs. 8, 11, and 12.

An interesting question is whether there is any shift V in $E_8 \times E_8$ [or $Spin(32)/Z_2$] which admits both spectra with and without five-branes. Such a situation could indicate possible transitions between perturbative and nonperturbative models which proceed through the emission of five-branes to the bulk [11].

Indeed, there is a unique case corresponding to the 'standard embedding,' $V = \frac{1}{3}(1, 1, 0, \ldots, 0) \times (0, \ldots, 0)$ [$V = \frac{1}{3}(1, 1, 0, \ldots, 0)$ for $Spin(32)/Z_2$], in which there are both a model without five-branes and a model with 18 five-branes. Both models have identical untwisted perturbative spectrum, but differ in that the twisted spectrum of the perturbative model has extra hypermultiplets with respect to the nonperturbative one.

In the $E_8 \times E_8$ case they transform as $(\mathbf{56}, \mathbf{1}) + 7(\mathbf{1}, \mathbf{1})$, while they organize as $(\mathbf{2}, \mathbf{28}) + 4(\mathbf{1}, \mathbf{1})$ under $SO(28) \times U(2)$ for $Spin(32)/Z_2$. The corresponding nonperturbative model contains just three singlets per fixed point in both cases. In the nonperturbative model the fixed points have magnetic charge $Q_f = -2$. This suggests that there can be transitions by which, around a fixed point in the perturbative model, these hypermultiplets go into two five-branes producing the nonperturbative model. The magnetic charge is conserved during the process since each fixed point has charge $Q_f = -2$ and each of the five-branes has charge $+1$.

In the $Spin(32)/Z_2$ case these transitions can be interpreted as an unhiggsing process where the rank is increased by two units, namely $(2, 28) + 4(1, 1) \rightarrow Sp(2) + matter$. If this transition occurs at each of the nine fixed points and all the branes are at the same (nonsingular point), an $Sp(18)$ maximum enhanced group is obtained, with the matter content specified in (1.12). A similar enhancing is expected to occur in $D = 4$.

The $E_8 \times E_8$ case is different. In the transition $(\mathbf{56}, \mathbf{1}) + 4(\mathbf{1}, \mathbf{1}) \rightarrow 2(\mathbf{1} + tensor)$ there is no enhancing at all and a complete charged hypermultiplet disappears into the bulk. In terms of M-theory branes this corresponds to an E_8 "fat" instanton, living on one of the "end of the world" nine-branes deflates, becoming pointlike, and going into the bulk as a five-M-brane. The separation between the nine-and the five-branes is given by the expectation value of the scalar field of the tensor multiplet on the five-brane [11, 13].

Interestingly enough, if an equivalent transition was possible in $D = 4$, for $N = 1$ it would imply a change in the number of generations. A chiral $\mathbf{27}$ generation (or $\overline{\mathbf{27}}$) of E_6, contained in the $\mathbf{56}$ of E_7, would disappear from the spectrum. We will show an explicit realization below.

Transitions can happen at each fixed point independently so that there should exist similar models with any even number of five-branes between 2 and 18. Thus, in these standard embedding models there is a discrete degree of freedom which corresponds to having pairs of zero-size instantons.

Here we have concentrated on a certain class of Z_3 models with enough blowing-up modes to resolve the singular points completely. A more general situation can be envisaged for cases where these modes are lacking ($V^2 > 8/9$ above) and for other Z_M orbifolds. This is extensively discussed in ref. 1. Let us just recall that generically nonsmooth models have five-branes trapped at these nonremovable singularities. The dynamics associated with these stuck branes is different from that of the smooth case. The behavior of such five-branes for the $SO(32)$ heterotic string is better known. It can be extracted from type I D-five-branes on ALE spaces and F-theory analysis [14–18]. For a larger enough number l_c of branes sitting at a singularity, further enhancements to unitary groups are expected. For instance, at a Z_3 orbifold point, $S_p(l) \times U(2l + m)$ is obtained [$l_c = (8 - m/2)$]. Tensor multiplets associated with the missing blowing-up modes do appear, somehow paralleling the $E_8 \times E_8$ case with wandering branes. Moreover, transitions where some hypermultiplets go into tensors are also suggested. For instance, when $m = 0$ and $l = 0$ in the Z_3 there is no enhancement at all and it is found that **28** + **1** $\rightarrow$ *tensor*, where **28** is a hypermultiplet transforming under a perturbative $U(8)$ [$\times SO(16)$] group. This parallels the above $E_8 \times E_8$ example.

The idea explored in the $D = 6$ case could be extended to $D = 4$, $N = 1$. One would construct heterotic orbifold vacua with perturbative and nonperturbative sectors in which the perturbative (but nonmodular invariant) sector could be understood in terms of simple standard orbifold techniques. We should also add a nonperturbative piece, but we face the problem that nonperturbative phenomena in $N = 1$, $D = 4$ theories are poorly understood at the moment. However, we can concentrate [1] on certain restricted classes of $D = 4$ orbifolds in which much of the structure is expected to be inherited from $D = 6$. In particular, one can consider $Z_N \times Z_M$ orbifolds in $D = 4$ with unbroken $N = 1$ supersymmetry. Such types of orbifolds have two general classes of twisted sectors, those that leave a 2-torus fixed and those that only leave fixed points. The first type of twisted sector is essentially 6-dimensional in nature; the twist by itself would lead to an $N = 2$, $D = 4$ theory, which would correspond to $N = 1$, $D = 6$ upon decompactification of the fixed torus. For this type of twisted sector we can use our knowledge of nonperturbative $D = 6$, $N = 1$ dynamics. Twisted sectors of the second type are purely 4-dimensional in nature and we would need extra information about 4-dimensional nonperturbative dynamics. To circumvent this lack of knowledge, one can restrict consideration to a particular class of $Z_N \times Z_M$ orbifolds with gauge embeddings such that these purely 4-dimensional twisted sectors are either absent or else are not expected to modify the structure of the model substantially.

Let us present a specific example [1] based on $E_8 \times E_8$. Consider the $Z_3 \times Z_3$ orbifold on $E_8 \times E_8$ with gauge shifts

$$\begin{aligned} A &= \tfrac{1}{3}(1, 1, 0, \ldots, 0) \times (0, \ldots, 0) \\ B &= \tfrac{1}{3}(0, 1, 1, 0, \ldots, 0) \times (0, \ldots, 0) \end{aligned} \tag{1.14}$$

This leads to a perfectly modular invariant orbifold with gauge group $E_6 \times U(1)^2 \times E_8$. However, we are going to consider the particular version of this orbifold with discrete torsion first considered in ref. 19. This model has the special property that all particles in the $(A + B)$ twisted sector are projected out. In this way we get rid of the sector which is purely 4-dimensional. The model has now three **27**'s in the untwisted sector and nine $\overline{\mathbf{27}}$'s in each of the sectors A, B, and $A - B$. Hence, altogether the model has 24 net antigenerations. We can now consider a nonperturbative orbifold in which the $D = 6$ subsectors A, B, and $A - B$ have a left-handed vacuum energy shifted by 1/3. This corresponds to a nonperturbative $D = 6$ vacuum with just singlets in the twisted sectors and 18 five-branes (leading to tensor multiplets) in each of the three twisted sectors.

Therefore, a transition from perturbative to the nonperturbative one implies

$$3(\mathbf{27}) + 27(\overline{\mathbf{27}}) \rightarrow 3(\mathbf{27}) \tag{1.15}$$

The 27 antigenerations of the twisted sectors disappear into the bulk and we are only left with three E_6 generations coming from the untwisted sector, plus singlets. The $U(1)$'s will now be anomalous, but there will be extra chiral singlets, coming from the tensors, with nonuniversal couplings to the gauge fields which will lead to a generalized version of the GS mechanism in $D = 4$.

Other examples undergoing chirality changes can be considered. A similar situation is found in $Spin(32)/Z_2$ when there are branes stuck at a fixed point [1]. A similar $Z_3 \times Z_3$ orbifold projection applied to the $U(8) \times SO(16)$ model mentioned above leads, for instance, to an $SU(6) \times SU(2)$ non-Abelian gauge group where a transition

$$(\overline{\mathbf{15}}, \mathbf{1}) + (\mathbf{6}, \mathbf{2}) \rightarrow \textit{singlets} \tag{1.16}$$

occurs. Notice that a nontrivial *anomaly-free* representation disappears from the spectrum.

Examples exhibiting chirality-changing transitions are particularly interesting. They correspond to $D = 6$ transitions in which one tensor multiplet transmutes into 29 charged hypermultiplets. Such sorts of transitions were also studied in ref. 23 from another approach. These examples show that the number of chiral generations is not invariant under nonperturbative effects, something inconceivable in perturbative field theory and also in perturbative string theory where the net number of generations is a topological number associated with a given compactified internal manifold. Vacua with a different number of generations can be connected. Even if the processes involve strong-coupling dynamics, quite presumably part or all the connected four-dimensional models can be realized perturbatively in some region of moduli space, thus effectively reducing the excessively huge vacuum degeneracy. In the explicit examples we have sketched above, these transitions may occur at each fixed point independently. If these are achieved at all nine Z_3 fixed points we end up with three generations and this number, associated with the untwisted sector of the orbifold, cannot be reduced further. In perturbative string theory some effort has been dedicated to finding appropriate compactifications leading to a small three, maybe four (nonvanishing) number of generations, hoping that nonperturbative physics would privilege these realizations over infinitely many others. These transitions indicate that, at least in some cases, a strongly coupled dynamics leading to models with few generations is available. Of course, other new phenomenological questions should be taken up now. For instance, since in other compactifications two or zero net generations are obtainable, which would be the preferred number? Another new, nonperturbative fact is that the gauge group may be significantly enhanced. This enhancement may be amazingly huge, and discouraging for predictivity, as was found in some very singular F-theory compactifications [24]. The situation is much more bounded in the models we have discussed above.

Most of the models we have constructed have candidate duals (this is a further check of our proposal) obtained from F-theory, M-theory, and type I string formulations [1]. Let us recall that our construction, even if formally feasible also in $D = 4$ dimensions, requires a better knowledge of nonperturbative effects. Even in $D = 6$, these effects are only partially known. As we stressed, only for a large enough number of branes $l \geq l_c$ on a fixed point and for a $Spin(32)/Z_2$ lattice is small instanton information available. This information is not yet available for $E_8 \times E_8$. The nonperturbative spectrum is not known in either lattice when the number of small instantons on the singularity is smaller than the critical value. The situation for $D = 4$, $N = 1$ vacua is even more uncertain. Some insight can be obtained from recent type IIB orientifold constructions [20, 21]. Moreover, we have seen that relevant nonperturbative information can be derived, in certain cases, from $D = 6$ physics.

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